Purpose: In this problem set, we will investigate end behaviors of rational functions. We will also explore polynomial long division, an important skill for analyzing end behavior of rational functions.

1. **Review:** Consider the rational function $f(x) = \dfrac{2x^2 + 4x + 2}{(x+1)(x-3)}$.

   (a) What is the domain of f?

   (b) If we stick to the domain of f, what is a simpler expression for f?

   (c) Does f have any vertical asymptotes? If so, where? What is the behavior of the graph of each side?

   (d) Does f have any holes? If so, where? (Your answer here should be a point.)

Just like with polynomials, we want to know what happens to rational functions as we plug in bigger and bigger x values.

Similar to polynomials, the end behavior relies on the leading terms, but <u>ratio</u>nal functions' end behavior relies on the <u>ratio</u> of the leading terms.

$$\text{Let } f(x) = \frac{A(x)}{B(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

There are three cases to check:

- The degree of the numerator is less than the degree of the denominator ($n < m$):

- The degree of the numerator is equal to the degree of the denominator ($n = m$):

- The degree of the numerator is more than the degree of the denominator ($n > m$):

2. Suppose $f(x) = \dfrac{3x^2 - 4x + 5}{x^3 + 4x^2 - 10}$. As $x \to \infty$, what does $f(x)$ tend to? As $x \to -\infty$, what does $f(x)$ tend to?

3. Suppose $f(x) = \dfrac{2x^3 - 82x}{-3x^3 + 10x^2 - 7}$. As $x \to \infty$, what does $f(x)$ tend to? As $x \to -\infty$, what does $f(x)$ tend to?

4. Polynomial long division note space:

5. Suppose $f(x) = \dfrac{2x^2 + 3}{x + 1}$. As $x \to \pm\infty$, what does $f(x)$ tend to?

Definition: The ______________________________ of a rational function is the line or polynomial that the rational function approaches in the long-run. When this feature is a line, we call it a ______________________________.

6. Use polynomial long division to simplify the following rational expression $\dfrac{x^4 + 3x^2 + x^3 + 2x + 2}{x^2 + 2}$.

Recap: Let

$$f(x) = \frac{A(x)}{B(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

- If $n < m$, then as $x \to \infty$, $f(x) \to 0$, and as $x \to -\infty$, $f(x) \to 0$.
- If $n = m$, then as $x \to \infty$, $f(x) \to \dfrac{a_n}{b_m}$, and as $x \to -\infty$, $f(x) \to \dfrac{a_n}{b_m}$.
- If $n > m$, $f(x)$ has asymptotic behavior determined by polynomial long division and the same end behavior as the monomial, $\dfrac{a_n}{b_m}x^{n-m}$. In particular, if $n = m + 1$, then $f(x)$ has an oblique (or slant) asymptote.